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ON LINDAHL'S THEORY OF DISTRIBUTION

by

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§ 1. Introduction :

Ever since Kaldor's seminal work on 'Alternative Theories of Distribution' (Kaldor, 1955) it has become customary to dichotomize research results - theoretical and empirical - in functional income distribution in terms of Neo-Classical vs. Neo-Keynesian (the latter also referred to as Kaldor-Pasinetti ; Post-Keynesian - even, at times, Neo-Ricardian) with appropriate footnotes referring to fringe theories ('degree of monopoly', Marxian, etc.) and distinguished predecessors (Boulding, Schneider, Hahn, etc.)[1] Indeed, in two stimulating articles recently published in this Journal (Brems, 1979) and Darity Jr. (1981) the underlying theme is in terms of variations on the above dichotomy.

It is, however, extremely surprising that Erik Lindahl's masterly contribution to this subject has been totally ignored. In what has now become a classic, as a precursor of General Equilibrium Dynamics, and the method of Temporary Equilibrium, the second, 'macroeconomic' part of Studies in the Theory of Money and Capital (Lindhal, 1939), contains, in a nutshell, the whole Neo-Keynesian theory of distributive shares (whether of the Kaldor version in terms of different types of income or of the Pasinetti version in terms of classes of people), as a special case with the complete version, using some dubious Austrian Capital Theory concepts, very much akin to the Robinsonian model of distribution (Robinson, 1956, 1962 ; cf. also Ferguson (1969) § 15.3.3. and Appendix to Chapter 15).

In this paper we attempt to substantiate these remarks by developing a simple model with well known concepts and references to the above work by Lindhal. Thus, in § 2, Lindahl's discussion is summarized. In § 3, a simple algebraic formulation

1. For example, Ferguson (1969) :
"So far as I can determine, the first 'alternative' theories of distribution based upon aggregate demand are attributable to Boulding and Hahn".
(p. 314)

of the model is proposed - but no explicit dynamics. We conclude, in § 4, with remarks on the relationship of the Lindahlian model to other strands in macro distribution theory and with some tentative observations for a reorientation of strategies in macroeconomic modelling suggested by Lindahl's work.

§ 2 . Lindahl on 'The Adjustment of Saving to Investment' :

It must, at the very outset, be pointed out that from the point of view of the virtues of the History of Economic Thought we should refer to Lindhal (1924) and Lindahl (1930) - particularly to the latter publication : Penningpolitikens Medel [1]. The discussions pertaining to the possibility of savings being brought to equality with investment by variations in the distribution of income are spelled out in great detail in Penningpolitikens Medel, and with particular emphasis. Much of this is abridged, and emphasized parts deleted or simply stated in the much too concise version in Part II of the English 'translation'[2].

However, since we expect that most of the interested readers will not be very proficient in Swedish - and, in any case, the two Swedish volumes are relatively inaccessible - or, in any of the other related Scandinavian languages,

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1. The preface to this book was signed by Lindahl in November 1929, and hence, is almost contemporaneous with Keynes (1930) and the "Widow's Cruse" parable which motivated Kaldor (but cf. Kaldor, 1978, p. ix., f.n. 1). Here, of course, the Wicksell link is clear (cf. also Wicksell, 1925, p. 216, f.n. 1, esp. the last two sentences and Wicksell (1898), esp. the end of chapter 9). But, of course, the idea really goes back to Ricardo's discussion of the high price of bullion cf. Ricardo's Appendix to The High Price of Bullion, published first in The Edinburgh Review, 1811, reprinted in Ricardo (1951), p. 99-127, esp. pp. 120-122.

2. For example, the crucial passage in the English version of Lindahl (1939) on p. 174 has an equivalent in the Swedish version in Lindhal (1930) p. 41. The reference to the mechanism of variations in the distribution of income leading to the savings-investment equality is in italics in the Swedish version (FÖRSKJUTNING AV INKOMSTFÖRDELNINGEN) which simply becomes 'redistribution of incomes' and deemphasized.

the references will only be to Lindahl (1939). It must not, however, be forgotten that for a complete and proper appreciation one must return to the Swedish original.

Lindahl poses the question of how savings will be brought to equilibrium with a higher level of investment, from an initial equilibrium situation, due to a lowering of the rate of interest which leads to 'increase in stocks' and (in a characteristically Austrian way) a 'reorientation of production in a more capitalistic direction (italics added, Lindahl 1939, p. 169). Thus, the seemingly paradoxical question is :

> "How can a lowering of the loan rate of interest which is generally supposed to have a tendency to decrease (voluntary) saving, thus cause an increase in total saving ?"
>
> (Lindahl, op. cit., p. 174)

Dismissing answers in terms of so-called 'forced saving' doctrines he goes on to make, what today would be characterized as a Neo-Keynesian or Kaldor-Pasinetti, the proposition that :

> "The solution appears to be, that while a lowering of the interest level might possibly diminish the propensity to save if the distribution of income were unaltered, it occasions a redistribution of incomes such that those with a relatively strong disposition to save find their incomes increased, at the expense of those whose disposition to do so is relatively weak".
>
> (Lindahl, op. cit., p. 174)

Lindahl concludes this particular section with the final remark that :

> "The required saving will then take place voluntarily in greater part [i.e., not by means of the mechanism of "forced saving"], and the causal element will

rather be the alteration in the distribution of income due to the shift in the price level".

(Lindahl, op. cit., p. 175)

In these two remarkable sections Lindahl proposes an original - for the period in question - approach to distribution - an approach almost validating Kaldor's remark to Hahn (cf. Hahn, 1973).

At a more formal level, Lindahl's model is very similar to the Robinsonian model of distribution with one important qualification (cf. below p. 6). Lindahl analyses the problem of the well known Wicksellian cumulative process under various alternative assumptions. The section of interest to us is characterized by the following assumptions :

a) The Economy is closed and in stationary (or steady) state.
b) Monetary Policy is autonomous.
c) There are two produced commodities :
   i) a consumption good
   and ii) a capital good
d) There are three classes :
   i) Workers
   ii) Capitalists (Rentiers in Mrs. Robinson's model)
   iii) Entrepreneurs (Capitalists in the Neo-Keynesian sense)
e) Two types of income : Wages and Profits
f) Full employment of all factors
g) No explicit role for a government
h) Given money wage rate
i) Given, constant, savings propensities of the three classes from the two types of income ; there is a clear indication that the savings propensities of the entrepreneurs are greater than those of the capitalists, and the latter's greater than the workers.
j) No cash holdings

It is not clear whether workers' savings propensity out of wage income is identical to those from profit income.

Under the assumptions stated above, Lindahl investigates the problem of the consequences to the cumulative process of a lowering of the rate of interest by the autonomous monetary authority ; and, as a by-product, stumbles on to the Neo-Keynesian or Kaldor-Pasinetti theory of functional income distribution.

There is, however, one crucial difference between Lindahl and 'mainstream' Kaldor-Pasinetti. That is the reliance on some features of a characteristically Austrian approach to the problem of investment in capital goods - and this is the point at which the similarity to Mrs. Robinson's model gets closest, and yet remains very different. It is not surprising that this similarity is in terms of the concept of 'more capitalistic' in terms of lower interest rates ; for, in Mrs. Robinson's model the corresponding concept was 'the degree of mechanization' in relation to the real-capital ratio which was defined in terms of the (notional) rate of interest. We read Chapter VI of Lindahl (1939) and numerous post-Sraffa articles by Joan Robinson (cf. in particular Robinson, 1975), to mean that they have both discarded and/or repudiated these dubious concepts [1]. For this latter reason we will not proceed to a formalization in a two-sector model where a lower rate of interest leads to investment in 'more capitalistic' (or 'increasing the degree of mechanization') methods and thus to price dynamics leading to alteration in the distribution of income. Instead, we formalize the Lindahl model in the traditional Kaldor-Pasinetti framework which, though not incompatible with a two-sector formulation in a

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1. There has been some criticism, paradoxically by Joan Robinson herself, of the reswitching and capital reversal results - particularly, the former. Her point is that 'switches' require comparison in time or over space. However she seems to have overlooked the peculiar assumption Sraffa (1960) made in § 93, in particular, pp. 82-84, to enable legitimate comparisons of two distinct systems. Sraffa, like Solow, may make peculiar assumptions but never makes mistakes - paraphrasing Sen (1974).

conventional way (cf. for example Ferguson's two-sector extension of the Kaldor model in Ferguson, op. cit., pp. 317-322 and Rothschild, 1971), does not need that particular framework and still is able to encapsulate the essential features of Lindahl's assumptions - as we will try to show in the next section.

Before proceeding to a simple, algebraic, formalization it must be pointed out that the variations necessary in the distribution of income, in Lindahl's model to equate savings and investment, is primarily brought about by 'the rising prices caused by the lowered interest rate' (Lindahl, op. cit. p. 174) - and finds its exact parallel in the Kaldor-Pasinetti system in the variation of profit margins as pointed out by Kaldor in his rejoinder to Tobin (cf. Kaldor, 1959-60).

§ 3. A Simple Lindahlian Model :

Notation [1] :

E = Entrepreneurs
C = Capitalists
L = Workers
W = Wages
P = Profits
Y = Output (Income)
S = Total Savings
I = Total Investment
s_{ep} = Savings Propensity of class E from Income P
s_{cp} = Savings Propensity of class C from Income P
s_{lw} = Savings Propensity of class L from Income W
s_{lp} = Savings Propensity of class L from Income P.

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1. The capitalists and entrepreneurs are 'transposed' from Lindahl's definition to conform to present terminology.

The following assumptions, in addition to those made in § 2, are implicit (or explicit, in some cases) in Lindahl's discussion :

$$s_{cp} > s_{ep} > s_{lp} > s_{lw} > 0 \qquad \ldots\ldots (1)$$

and, we denote by K_c , K_e and K_l the part of the total stock of capital owned by the three classes respectively. Similarly, P_c , P_e and P_l denote the part of total profits accruing to the three classes. Thus, we have a three-class - two income closed economy, without a government, (producing two goods), in a stationary (or steady-state) equilibrium. Total savings are given by :

$$S = s_{ep} P_e + s_{cp} P_c + s_{lw} W + s_{lp} P_w \qquad \ldots\ldots (2)$$

and $$P_e + P_c + P_w = P \qquad \ldots\ldots (3)$$

also $$Y = W + P \qquad \ldots\ldots (4)$$

gives $$\frac{W}{P} = 1 - \frac{P}{Y} \qquad \ldots\ldots (5)$$

Then from simple substitution and the Savings-Investment Equality, in equilibrium, we get :

$$\left(s_{ep} - s_{lp}\right)\frac{P_e}{Y} + \left(s_{cp} - s_{lp}\right)\frac{P_c}{Y} + \left(s_{lp} - s_{lw}\right)\frac{P}{Y} = \frac{I}{Y} - s_{lw} \qquad \ldots (6)$$

and $$\left(s_{ep} - s_{lp}\right)\frac{P_e}{K} + \left(s_{cp} - s_{lp}\right)\frac{P_c}{K} + \left(s_{lp} - s_{lw}\right)\frac{P}{K} = \frac{I}{K} - s_{lw}\frac{Y}{K} \qquad \ldots (7)$$

These collapse to Kaldor's or Pasinetti's systems when the appropriate assumptions are made.

Assuming that the rate of interest continues to be equal to the rate of profit and that the proportion of capital held by each class is identical with the proportion of total savings made by that class, it is easy to derive the following :

$$\frac{P}{Y} = \frac{1}{s_{cp}} \frac{I}{Y} \qquad \ldots\ldots (8)$$

and $$\frac{P}{K} = \frac{1}{s_{cp}} \frac{I}{K} \qquad \ldots\ldots (9)$$

These are, indeed, Pasinetti's celebrated results - and are implied by the Lindahl system. Thus, we can see, trivially, that given the investment-income ratio the share of profits adjusts in such a way that savings are equated with investment. But the problem is that Lindahl does not assume an exogenously given investment-income ratio - instead, investment is 'endogenized' almost identifically to the Robinsonian model in Ferguson (op. cit.) § 15.3. For reasons that have been mentioned above we refrain from pursuing this path. To investigate the problem of the adjustment of savings to investment when the rate of interest is lowered, in relation to the rate of profit obtained by capitalists, we proceed, once more, along the lines suggested by Pasinetti :

Denote by $\dot{i}$, the rate of interest accruing to the entrepreneurs and workers. In equilibrium, then :

$$\dot{i} = \frac{P_e}{K_e} = \frac{P_w}{K_w} \qquad \ldots\ldots (10)$$

where $\dot{i}$ is the rate of interest.

This rate is lowered in relation to the rate of profit obtained by capitalists on their capital, which is, of course, $\frac{P_c}{K_c}$.

Then following Pasinetti (1974), p. 140 ff, we can put :

$$\dot{i} = \mu \frac{P_c}{K_c} \quad \text{where } \mu < 1 \qquad \ldots\ldots (11)$$

and therefore :

$$\frac{P}{Y} = \frac{1}{\gamma s_{cp}} \frac{I}{Y} \qquad \ldots\ldots (12)$$

$$\frac{P}{K} = \frac{1}{\gamma s_{cp}} \frac{I}{K} \qquad \ldots\ldots (13)$$

Where, for the equilibrium growth at the natural rate, we get :

$$\gamma = f\left(g_n\ ,\ s_{cp},\ s_{ep},\ s_{lp},\ s_{lw},\ \mu\ ,\ k^*,\ \lambda_i\right) \quad \ldots\ldots\ (14)$$

where k^* : equilibrium capital-output ratio

g_n : the equilibrium growth at the natural rate (Note : Lindahl assumes full employment, steady state).

and $\gamma = 1$ when $\mu = 1$ or all savings propensities except s_{cp} equal to zero.

and $\gamma > 1$ when $\mu < 1$ and the savings propensities > 0.

λ_i : the proportion of profit-earners in each category (the complement, in a sense, of Pasinetti's λ_i in f.n. 25, p. 142, Pasinetti, op. cit.).

Thus, it is clear that a rate of interest less than the rate of profit ($\mu < 1$) is equivalent to augmenting the savings propensity of the capitalists - or, what comes to the same thing, changing the distribution of income in favour of those with high savings propensities.

Although the distribution of income, between the share of profits as a whole and, therefore, the wage share, is determined given the investment-income ratio, the distribution of profits between classes is not. To determine this we proceed as follows :
In equilibrium (even when the autonomous rate of interest is less than the rate of profit), we must have :

$$\frac{P_w}{P_e} = \frac{s_w}{S_e} = \frac{s_{lw}\ W + s_{lp}\ P_w}{s_{ep}\ P_e} \quad \ldots\ldots\ (15)$$

Then, $$\frac{P_w}{W} = \frac{s_{lw}}{(s_{ep})^2 - s_{lp}} \quad \ldots\ldots\ (16)$$

and, therefore

$$\frac{P_w}{Y} = \frac{P_w}{W} \cdot \frac{W}{Y} = \left[\frac{s_{lw}}{(s_{ep})^2 - s_{lp}}\right]\left(1 - \frac{P}{Y}\right) \quad ....(17)$$

Thus, from (12), (16) and (17) we can determine the distribution of income between Profits and Wages, on the one hand, and between workers and the rest ; however, the distribution of profit income between capitalists and entrepreneurs still remains to be determined. At equilibrium the proportion of capital owned should be equal to the proportion of savings, and for the entrepreneurs, we get :

$$\frac{K_e}{K} = \frac{S_e}{S} = \frac{s_{ep} P_e}{I} \quad ....(18)$$

writing $$\frac{P_e}{Y} = \frac{P_e}{K} \cdot \frac{K}{Y} \quad ....(19)$$

and using the result on the proportion of capital stock owned by each class which we have already done to determine γ (cf. f.n. 25, p. 142 in Pasinetti -op.cit.-) we get :

$$\frac{P_e}{K} = \cdot \frac{g_n}{s_{ep}} \cdot \frac{K_e}{K} \quad ....(20)$$

and hence

$$\frac{P_e}{Y} = \frac{g_n}{s_{ep}} \cdot k^* \cdot \frac{K_e}{K} \quad ....(21)$$

where $\frac{K_e}{K}$ is a function of the parameters of the system.

Therefore, using (6), (7), (12), (16), (17) and (21) we can determine the distribution of profits between classes.

From these results it is clear that the Lindahl model is conceptually richer than any of the classic Kaldor-Pasinetti models, but is very much in that tradition if we abstract from the unsound features of tenuous (Austrian) capital

theoretic concepts. It is, of course, possible to proceed along the alternative modelling route of an explicit two-sector model and introducing a concept analogous to the Robinsonian (-Wicksellian) concept of a degree of mechanization related (monotonously) to the notional rate of interest and its deviation from the even more dubious concept of a natural rate of profit. This latter concept was explicitly repudiated also by Lindahl.

§ 4. Concluding Notes :

The real richness of a Lindahlian model would be the next step : from an equilibrium (stationary or steady-state) within periods to the transition from one equilibrium to another. It is the problem of traverse - in Hicks's terminology - or the problem of disequilibrium dynamics in the terminology of modern macrodynamics. The roots from which the problem of traverse and the problem of disequilibrium dynamics spring are, of course, related : Hicks's reformulation, on the one hand, of Austrian capital theory, and, on the other hand, also in the hands of Hicks, the revitalization of the temporary equilibrium method of the Stockholm school - in particular Lindahl, Myrdal and Svennilsson. These, in turn, go back to the problem of a proper formulation of Social Accounting Relations (cf. Hicks, 1973 , in particular Chapter III, and Hicks, 1956).

Is the hint, then, that, if we are to go beyond steady state dynamics and attempt the difficult problems of disequilibrium dynamics along the traverse the proper starting place is 'macropolitics' (Hicks, 1977), and Social Accounting via Public Finance - i.e., the dynamics of a Political Equilibrium. These are not too different from the reasons given by Kaldor, in his rejoinder to Tobin, for the method of macrodynamics he was following.

Though it is now customary to dismiss (as hand-waving) models in which accounting identities are manipulated to understand implicit dynamics, it is not without significance that the two most promising approaches to proper disequilibrium dynamics can be traced back to the works of the Swedes who never tired of seeking the fundamentals of dynamics and disequilibria in the balance-sheet of the macroeconomy.

Naturally, to remove the strait-jacket of steady-state dynamics, or Golden Age condition, we must introduce dynamics in the constituent functions. At this point we can proceed along conventional neo-classical lines (for eg. as in Stiglitz, 1967) or attempt a new approach along Neo-Keynesian or Neo-Cambridge lines. In the latter case the starting points would be one of the many Kaldorian growth models of the '50's and early '60's, or models of the Kalecki or Goodwin type. Nothing less than analysis in terms of Differential Games would be sufficient in this case and that is not a trivial task.

However, it is extremely significant that both Kaldor and Lindahl arrived at their interesting macro-economic theories of distribution from simple social accounting systems. Political Arithmetic is, after all, the very foundation of our subject.

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